1. $\sin \frac{\pi}{6} = \frac{1}{2}$

2. $\cos \frac{\pi}{3} = \frac{1}{2}$

3. $\tan \frac{\pi}{4} = 1$

4. $\sec \frac{\pi}{3} = 2$

5. $\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$

Apr 4-8:29 AM

21. $f(x) = 9x^{2/3} + 3x - 6$

max/min

critical pts { $f' = 0$, $f' = \cancel{\exists}$ }

endpts

$f'(x) = \frac{2}{3} \cdot 9x^{-\frac{1}{3}} + 3$

$0 = \frac{6}{\sqrt[3]{x}} + 3$

$x = 0$

$-3 = \frac{6}{\sqrt[3]{x}}$

$\sqrt[3]{x} = -2$

$x = -8$

$f'' = 6\left(-\frac{1}{3}\right)x^{-\frac{4}{3}}$

min at $x = 0$

f': $+$ 0 $-$ $\ast$ $+$ at -8, 0

Apr 4-8:50 AM

22.

x	f	f'
0	2	5
4	-3	11

$\int_0^4 f(x)\,dx = 8$

$\int$ product : parts

$\int_0^4 x\, f'(x)\, dx$

$u = x \quad du = dx$

$f'(x)\,dx = dv$

$v = f(x)$

$x f(x)\Big|_0^4 - \int_0^4 f(x)\,dx$

$\left[4 f(4) - 0 \cdot f(0)\right] - 8$

$4(-3) - 8 = -20$

Apr 4-8:57 AM

24.

$A = \pi r^2 \qquad C = 2\pi r$

$\frac{dA}{dt} = 2\,\frac{dC}{dt} \qquad$ find r

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} \qquad \frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\cancel{2}\pi r \cancel{\frac{dr}{dt}} = 2 \cdot \cancel{2}\pi \cancel{\frac{dr}{dt}}$

$r = 2$

Apr 4-9:03 AM

26.

$$\sum_{n=0}^{\infty} a_n (x-2)^n \qquad a_0 = 5 \qquad a_n = \left(\frac{2n+1}{3n-1}\right) a_{n-1}$$

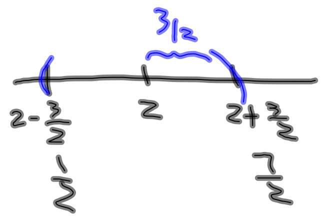
ratio test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}(x-2)^{\cancel{n+1}}}{a_n \cancel{(x-2)^n}} \right| \qquad \frac{a_n}{a_{n-1}} = \frac{2n+1}{3n-1}$$

$$\lim_{n\to\infty} \left| \frac{2n+1}{3n-1}(x-2) \right| = \frac{2}{3}|x-2| < 1$$

$$|x-2| < \frac{3}{2}$$

center radius



Apr 4-9:08 AM

27.

$$f(x) = \int_4^{2x} \sqrt{t^2 - t}\; dt \qquad f'(2) =$$

$$f'(x) = \sqrt{(2x)^2 - (2x)} \cdot 2 \Big|_{x=2}$$

$$= \sqrt{16-4} \cdot 2$$

$$= \sqrt{12} \cdot 2$$

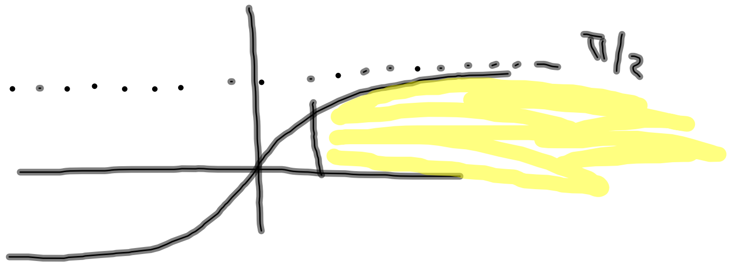
Apr 4-9:14 AM

Review 21 Improper Integrals

$$\int_1^{\infty} \tan^{-1} x \, dx$$

infinity as a limit of integration

fix: $\lim_{b\to\infty} \int_1^b \tan^{-1} x \, dx = \infty$ diverges

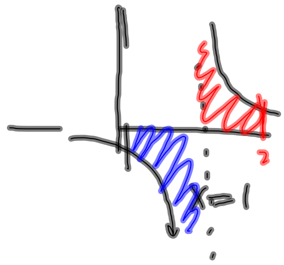


Apr 4-9:18 AM

$$\int_0^2 \frac{1}{x-1} \, dx$$

Improper at $x=1$

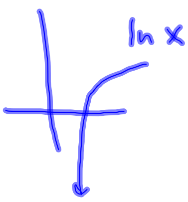
$$\overset{\text{diverges}}{\int_0^1 \frac{1}{x-1} \, dx} + \int_1^2 \frac{1}{x-1} \, dx$$



$$\lim_{b\to 1} \int_0^b \frac{1}{x-1} \, dx$$

$$\lim_{b\to 1} \ln|x-1| \Big|_0^b = \lim_{b\to 1} \ln|b-1| - \ln|-1|$$

$$= -\infty + 0$$



Apr 4-9:22 AM

does the integral converge or diverge

$$\int_1^\infty \frac{1}{1+x^3}\,dx < \int_1^\infty \frac{1}{1+x^2}\,dx$$

direct comparison test

conv by comparison

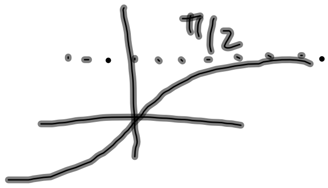
conv.

or $< \int_1^\infty \frac{1}{x^3}\,dx$

$$\lim_{b\to\infty} \tan^{-1}x \Big|_1^b$$

$$\lim_{b\to\infty} \tan^{-1}b - \tan^{-1}1$$

$$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$



Apr 4-9:28 AM

limit comparison test

$$\int_1^\infty f(x)\,dx \quad \& \quad \int_1^\infty g(x)\,dx$$

behave the same if $f(x)$ & $g(x)$ grow at the same rate

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$$

$$0 < L < \infty$$

$$\lim_{x\to\infty} \frac{\frac{1}{1+x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{x^3}{1+x^3} = 1$$

Apr 4-9:33 AM